Error statistics in data transmission over fading channels

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***Abstract*—In this paper, we investigate the behavior of block errors which arise in data transmission on fading channels. Our approach takes into account the details of the specific coding/modulation scheme and tracks the fading process symbol by symbol. It is shown that a Markov approximation for the block error process (possibly degenerating into an iid process for sufficiently fast fading) is a good model for a broad range of parameters. Also, it is observed that the relationship between the marginal error rate and the transition probability is largely insensitive to parameters such as block length, degree of forward error correction and modulation format, and depends essentially on an appropriately normalized version of the Doppler frequency. This relationship can therefore be computed in the simple case of a threshold model and then used more generally as an accurate approximation. This observation leads to a unified approach for the channel modeling, and to a simplified performance analysis of upper-layer protocols.**

***Keywords*—Error statistics, Markov model, Gilbert channel, Rayleigh fading, Error correlation.**

I. Introduction

Interest in wireless communication has increased in recent years, primarily to foster Personal Communication Services (PCS). In this context, there is a greater demand for capacity and a need to integrate voice, data and other types of traffic over radio channels. One of the key technical problem areas that has emerged out of this is the need to determine the effect of fading on network performance. One specific problem that arises in this context is the performance analysis of data-link protocols.

In the literature, most models that have been used for this purpose assumed that the block transmissions were independent and identically distributed (iid). Also, many protocols and coding schemes were designed for an iid channel, and techniques were developed to eliminate channel memory (e.g., interleaving). An alternative approach may be to take advantage of the channel memory (e.g., by exploiting some prediction techniques) to obtain better performance. In fact, turning a channel with memory into a memoryless one by interleaving is not necessarily an efficient way of using it, since the interleaving operation may substantially reduce the channel capacity [1; 2].

Thus, the study of channels with memory is important. However, some real-world channels do not lend themselves to an analytical study, and the development of simplified models is desirable. A natural approach is to approximate a channel with memory by means of a Markov model [3]. In [4], Wang investigated the accuracy of a first-order Markov process in modeling transmission on a flat Rayleigh fading channel, finding that such an approximation is, in fact, satisfactory. The process studied is the sequence of values of the envelope of the complex Gaussian process used to model the multiplicative effect of the channel. Markov models to approximate the Rayleigh fading channel have also been considered in [2; 5].

However, quite often it is not the value of the channel envelope which is of direct interest, but rather some nonlinear function of it, which depends on the transmission techniques, modulation, coding, and so on. In particular, the binary process which describes the successes and the failures of the block transmissions is of primary importance, and very often is the only information which is available.

The binary process which describes the success/failure of the data block transmissions has been considered by Zorzi *et al.* [6] for a simple channel model in which a block is successfully received if and only if the value of the fading envelope is above a certain threshold. We shall refer to this as the "threshold model" in the sequel. Using mutual information, and following an approach similar to that used in [4], it was shown that such a process can be adequately approximated by a binary Markov process, which corresponds to a Gilbert channel model[1] [8]. However, the simple threshold model (which incorporates the effect of all the details of the coding/modulation scheme into a single parameter, i.e., the threshold value), may not always be realistic. For example, the threshold model implicitly assumes that the value of the fading envelope is constant throughout the duration of a data block, and this might not be true in the presence of a significant degree of mobility.

In this paper, we extend the approach in [6] by including some actual coding/modulation schemes, and by tracking the fading envelope symbol by symbol. This detailed approach includes simulation of the third-order statistics of the block error process. It is shown that, for various modulation schemes, and for various block lengths and error correction capabilities, the Markov model for block errors is very good. Also, when the fading is fast enough, the Markov model degenerates into an iid process, as expected. Finally, it is shown that the relationship between the two Markov parameters is largely insensitive to the system parameters considered (block length, error correction capability and modulation scheme), and depends essentially on an appropriately normalized version of the Doppler frequency. This relationship is, in fact, almost the same as in the simple threshold model, for which closed-form expressions were obtained in [6] (for the Rayleigh fading case). This fact suggests a unified approach to the modeling of the physical channel, which could

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[1] A similar approach, in which the channel memory is captured by means of a Markov model at the bit level, has also been followed in [7].

simplify the analysis of upper layer protocols.

The paper is organized as follows. In Section II, the fading model is described. In Section III, the success/failure process of data block transmissions is introduced, and its accuracy is discussed. The usefulness of the threshold model as an approximation in more general situations is discussed in Section IV, and an example application is presented in Section V.

II. FADING MODEL

We assume a flat fading channel, modeled as a multiplicative complex function, $\alpha(t)$, which is adequately described as a random process. A popular model considers a complex Gaussian random process with a given mean and covariance function [9]. On the time scale of the fading variations, the process can be considered as stationary. Therefore, with no loss in generality, we will normalize its power to 1. The real and imaginary axes can be chosen so that the mean, $\mu = E[\alpha(t)]$, is real. Also, we consider the covariance function, defined as

$$K(\tau) = E[(\alpha(t+\tau) - \mu)^*(\alpha(t) - \mu)]. \qquad (1)$$

Note that if $\mu = 0$, the envelope of $\alpha(t)$ is Rayleigh distributed for any t, and the envelope squared has an exponential probability distribution. On the other hand, when $\mu > 0$, we have the *Rician fading* model, which accounts for the presence of a line-of-sight (LOS) component, and is often more accurate in micro- and picocells. When the LOS component is absent, or has negligible power, the Rician model degenerates into the Rayleigh one.

In a widely accepted model, the Gaussian process is assumed to have a bandlimited non-rational spectrum, given by [9]

$$S(f) = S(0)\left[1 - \left(\frac{f}{f_D}\right)^2\right]^{-1/2}, \quad \text{for } |f| < f_D, \qquad (2)$$

and zero otherwise. This spectrum corresponds to the covariance function

$$K(\tau) = J_0(2\pi f_D|\tau|), \qquad (3)$$

whose physical meaning has been investigated in [9; 10]. In (3), $J_0(\cdot)$ is the Bessel function of the first kind and of zeroth order, and f_D is the Doppler frequency, equal to the mobile velocity divided by the carrier wavelength [9]. Note that the correlation properties of the fading process depend only on $f_D|\tau|$. When $f_D|\tau|$ is small (< 0.1), the process is very correlated ("slow" fading); on the other hand, for larger values of $f_D|\tau|$ (> 0.2), two samples of the channel are almost independent ("fast" fading).

Note also that, for high data rates (small τ), the fading process can typically be considered as slowly varying, at least for the usual values of the carrier frequency (i.e., 900-1800 MHz) and for typical vehicular speeds, in the sense that the dependence between transmissions of consecutive blocks of data cannot be neglected. In particular, the assumption that the successes/failures of data blocks constitute an iid process is unrealistic, and may lead to incorrect results when used to evaluate the performance of a transmission scheme or protocol. A more general model for the success/failure process, which accounts for dependence, is developed in the next section.

III. THE BLOCK ERROR PROCESS

Let $\alpha_i = \alpha(iT)$ be a discrete version of the fading process, obtained by sampling the channel at the symbol rate (i.e., every T seconds), and let $v_i = |\alpha_i|$ be the corresponding value of the fading envelope. For even moderately high data rates (e.g., in excess of 64 kb/s), and for the environment commonly considered (i.e., carrier frequency in the vicinity of 1-2 GHz, and pedestrian or vehicular speeds), we have $f_D T \ll 1$. Therefore, assuming that the channel is constant during a symbol interval is reasonable [11]. On the other hand, this might not be true over the duration of a data block, which usually spans a number of symbols, N, that may be several tens up to a few thousands.

To study the block error process, we define a binary sequence, β_i, where $\beta_i = 0$ if a block is correctly decoded, and $\beta_i = 1$ if an undecodable error is detected. For simplicity, we ignore here the occurrence of undetected errors, whose probability can be made negligible (especially compared with typical values of the block error rate in the wireless environment) by a suitably chosen error-detection code.

An interesting issue is how to effectively model the process β_i in a simple manner. In [6], Zorzi *et al.* considered a simple threshold model such that [12]

$$\beta_i = \begin{cases} 0 & \text{if } v_{Ni}^2 > b, \\ 1 & \text{if } v_{Ni}^2 \le b, \end{cases} \qquad (4)$$

where b is the power threshold, $F = 1/b$ is the *fading margin*, and "1" stands for a block failure. The value of the fading envelope during the first symbol, v_{Ni}, is implicitly taken to represent the fading values throughout the whole block, $(v_{Ni}, v_{Ni+1}, \ldots, v_{N(i+1)-1})$. The value of the threshold parameter, b, accounts for the dependence of the model on the details of the modulation and coding scheme used. It was shown in [6] that, for this channel model, an adequate approximation for the process β_i is a binary Markov process, for which the knowledge of the conditional probability matrix is sufficient [3]. In [6], b was assumed to be known. In practice, this information may not always be readily available. This issue is further discussed in Section IV.

The threshold model, although easy to study [6], appears to be an oversimplification of more realistic schemes (in which, for example, the fading envelope might vary significantly during a block). Therefore, a more detailed investigation, taking into account all the details of the modulation format, is desirable.

Let γ_j be a binary process such that $\gamma_j = 1$ if symbol j is in error and 0 otherwise. That is,

$$\gamma_j = \begin{cases} 1 & \text{with probability } P_e(v_j) \\ 0 & \text{with probability } 1 - P_e(v_j), \end{cases} \qquad (5)$$

where P_e is the conditional symbol error probability (given the value of the fading envelope, v_j), which also depends on the modulation scheme used and on the average signal-to-noise ratio on the channel.

We assume that the stream of symbols is subdivided into blocks, each composed of k symbols. We assume here that an (N, k) block code is used, which maps each block of k symbols into an N-symbol codeword, and which is able to correct all patterns of

n_e or fewer errors in a block. In this case,

$$\beta_i = \begin{cases} 1 & \text{if } \sum_{j=(i-1)N+1}^{iN} \gamma_j > n_e \\ 0 & \text{otherwise,} \end{cases} \tag{6}$$

where we assume that a block is correctly received if it contains n_e or fewer symbol errors, and is detected in error otherwise. Undetected errors are ignored for simplicity.

In this paper, we will consider two modulation schemes relevant to the mobile radio environment. The first one is Binary Phase-Shift Keying (BPSK) with coherent detection, which is also equivalent to QPSK (in the absence of crosstalk). Given the value of the fading envelope, $v = |\alpha|$, the conditional probability of a bit error is in this case given by [13, p. 246]

$$P_e(v) = \frac{1}{2}\text{erfc}\left(v\sqrt{\lambda}\right), \tag{7}$$

where $\lambda = E_b/N_0$ is the mean signal-to-noise ratio per information bit, and

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt, \qquad x \geq 0, \tag{8}$$

is the complementary error function.

The second modulation scheme considered here is orthogonal signaling (e.g., FSK) with non-coherent detection. The alphabet is M-ary, i.e., each symbol carries ν information bits, with $M = 2^\nu$. The symbol error probability, conditioned on the value of the fading envelope, v, is given by [13, p. 296]

$$P_e(v) = \sum_{n=0}^{M-1} (-1)^{n+1} \binom{M-1}{n} \frac{1}{n+1} e^{-n\nu v^2 \lambda/(n+1)}, \tag{9}$$

where, again, λ is the signal-to-noise ratio per information bit. A special case is $M = 2$, where

$$P_e(v) = \frac{1}{2} e^{-v^2\lambda/2}. \tag{10}$$

Note that (10) also gives the performance of binary DPSK, by removing the factor $1/2$ in the exponent.

A. Modeling the success/failure process

In this subsection, we consider the issue of whether or not the error process can be modeled as Markov. It will turn out that this is an adequate approximation, both for slow fading (successive samples are very correlated) and for fast fading (successive samples are almost independent).

Let $I(\beta_i;\beta_{i-1}\beta_{i-2})$ be the average mutual information between the r.v. β_i and the past two transmissions, β_{i-1} and β_{i-2}. We can write [4]

$$I(\beta_i;\beta_{i-1}\beta_{i-2}) = I(\beta_i;\beta_{i-1}) + I(\beta_i;\beta_{i-2}|\beta_{i-1}), \tag{11}$$

where $I(\beta_i;\beta_{i-1})$ is the average mutual information between β_i and β_{i-1}, and $I(\beta_i;\beta_{i-2}|\beta_{i-1})$ is the average mutual information between β_i and β_{i-2}, once β_{i-1} is known. A measure of the goodness of the first-order Markov approximation can be given in terms of

$$\zeta = \frac{I(\beta_i;\beta_{i-2}|\beta_{i-1})}{I(\beta_i;\beta_{i-1})}. \tag{12}$$

That is, if $\zeta \ll 1$, the relative importance of the numerator is small with respect to the denominator, meaning that, after β_{i-1} is known, the additional information on β_i carried by β_{i-2} is negligible.

This approach was used by Wang [4] with reference to the values of the fading envelope, $v_i = |\alpha_i|$. Here, as in [6; 7], we will apply the same technique to the success/failure process, β_i. In principle, other techniques are possible, e.g., a statistical test on the geometric distribution of the dwell times in each state, although collecting enough data for a meaningful statistical analysis appears in this case very hard. We note that passing one of these statistical tests (mutual information or geometric distribution) does not by itself guarantee that the process is Markovian, as these properties are necessary but not sufficient conditions. Therefore, assessment of the Markov character of a process based on such statistical tests is to be regarded as an approximation. The goodness of this approximation is confirmed by the excellent match between performance results based on the Markov error model and those obtained by direct simulation of the block transmissions over the fading channel (an example is given in Section V).

The analytical evaluation of the third-order statistics of β_i, while possible for the threshold model (see [6] for details), appears impossible to perform in the present context, where the actual coding/modulation scheme is taken into account and the fading envelope is allowed to vary symbol by symbol. Therefore, we compute the statistics of β_i by simulation.

For a given value of the normalized Doppler frequency, $f_D T$, the fading process was simulated according to the technique proposed by Jakes [9], and sampled at the symbol rate. Here we assume that the fading variation within a symbol duration is negligible (see [11] for a discussion). For a given modulation format and for a given value of the mean signal-to-noise ratio, λ, the error probability, $P_e(v)$ (as given in (7) and (9)), was computed for each symbol, conditioned on the value of the fading envelope, v, during that symbol. (Values of λ ranging from 0 to 20 dB were considered, which cover most practical situations.) Given the values of v for each symbol and the corresponding error probabilities, errors on distinct symbols occur independently, and can be easily simulated. Finally, the symbols were grouped into N-symbol blocks, and a block error was declared if more than n_e errors were observed within a block. Each simulation involved 250,000 blocks, corresponding to 500 million symbols for the largest block size considered in our computations ($N = 2000$). From these simulations, the complete third-order statistics of β_i were estimated.

B. Accuracy of the Markov model for β_i

In Figures 1 and 2, we plot the entropy of the binary process β_i, $H(\beta)$, the average mutual information, $I(\beta_i;\beta_{i-1})$, and the conditional average mutual information, $I(\beta_i;\beta_{i-2}|\beta_{i-1})$, vs. the average block error rate, for BPSK and M-ary FSK, respectively. The parameter $f_D T$ (the Doppler frequency normalized to the data rate) is taken equal to 0.0001. If 100 kb/s is taken as a reference value for the channel data rate, so that $T = 10\mu$s, this corresponds to a Doppler frequency of 10 Hz, i.e., at 900 MHz, to a mobile speed of about 3 m/s, which represents a medium/low degree of mobility.

Figure 1 shows the results for BPSK. It can be seen that $I(\beta_i;\beta_{i-2}|\beta_{i-1})$ is typically at least one order of magnitude smaller than $I(\beta_i;\beta_{i-1})$, for the considered values of the parameters. The only exception is the case with block length $N = 2000$ bits, for which both $I(\beta_i;\beta_{i-2}|\beta_{i-1})$ and $I(\beta_i;\beta_{i-1})$ are $\ll H(\beta)$. Under these conditions, two channel samples NT seconds apart (i.e., at "block distance"), have a correlation coefficient of $J_0(2\pi f_D NT)$. When $f_D NT$ gets large (say, larger than 0.1), the block errors approach an iid process, as demonstrated by the fact that $I(\beta_i;\beta_{i-1}) \ll H(\beta)$. In all cases, the Markov approximation for β_i appears to be a reasonable model.

Figure 2 shows that similar results can be obtained for noncoherent M-ary FSK modulation, and are largely insensitive to the alphabet size, M. In particular, different values of M result in perfectly overlapping curves for $H(\beta)$ and $I(\beta_i;\beta_{i-1})$. Also, entirely similar results (not shown here) have been obtained for other code parameters (i.e., for different values of the error correction capability of the code, n_e), confirming the validity of the above findings and their wide applicability.

In Figure 3, we investigate the dependence on the Doppler frequency. Results similar to those of Figure 1 are plotted for block length $N = 500$ bits and for $f_D NT = 1, 0.1$ and 0.01, corresponding (for 100 kb/s and 900 MHz) to a Doppler frequency (mobile speed) of 200 Hz (66 m/s), 20 Hz (6.6 m/s) and 2 Hz (0.66 m/s), respectively. As expected, for fast fading ($f_D NT = 1$ in this example), the error process is almost iid (i.e., $I(\beta_i;\beta_{i-1}) \ll H(\beta)$), whereas for slower fading the Markov approximation is reasonably accurate.

C. Markov parameters

We now study how the system parameters affect the Markov description, which is defined by the transition matrix

$$M(x) = \begin{pmatrix} p(x) & q(x) \\ r(x) & s(x) \end{pmatrix} = \begin{pmatrix} p & q \\ r & s \end{pmatrix}^x, \qquad (13)$$

where $p(x) = 1 - q(x)$ (or $r(x) = 1 - s(x)$) is the probability that the i-th block is successfully transmitted, given that the $(i-x)$-th block was successful (or unsuccessful). Note that the model is completely specified by two independent parameters. Following most of the literature, we will consider here the transition probability $r = P[\text{success}|\text{failure}]$, and the steady-state error rate, ε, given by

$$\varepsilon = \frac{q}{q+r}. \qquad (14)$$

The quantities ε and r can be evaluated from the second-order joint distribution computed by simulation, and have their own physical meaning, regardless of the error process statistics. However, their significance is greatly increased in the presence of a Markov error process, since they are sufficient for its complete characterization.

In Figures 4 and 5, we plot the average length of a burst of block errors, $1/r$, vs. the average block error rate, ε, for block length $N = 500$ symbols and for various values of $f_D NT$. The results for the threshold model (discussed in the next section) are also plotted for comparison.

Figure 4 shows that the relationship between r and ε is relatively insensitive to the code parameters for constant value of the block size, N. For clarity, only $n_e = 4$ and 50 are shown in the figures, but the curves obtained for other values of n_e are clustered together. Figures 4 and 5 also indicate that the relationship between ε and r is insensitive to the value of M in M-ary FSK (see Figure 5), and even to the modulation format (compare Figures 4 and 5).

As one could have expected, for a given value of the average error rate, ε, the average length of a burst of errors increases as the normalized Doppler frequency decreases (i.e., the fading process is slower). However, from Figure 4 it can be seen that, for practically acceptable values of the error rate (say, not exceeding 10%), even for fairly small values of $f_D NT$, the average burst length is less than ten slots.

Quantitative knowledge of the statistics of the number of consecutive slots in error can help tune parameters such as timeout counters or retransmission timers in order to exploit the channel information. Also, since the average burst length is given here as a function of the average error rate, a communication scheme could even automatically adapt its parameters, according to these relationships, based on real-time estimates of the channel conditions.

IV. APPLICABILITY OF THE THRESHOLD MODEL

Based on the results reported above, two main conclusions can be drawn. First of all, a binary Markov model for the block error process, β_i, is an adequate approximation for a broad range of the parameters (see Figures 1 through 3). Second, for a block size of $N = 500$, the relationship between the two Markov parameters, the average burst length, $1/r$, and the average error rate, ε, is relatively insensitive to the coding/modulation scheme used (see Figures 4 and 5). This relationship is essentially determined by an appropriately normalized version of the Doppler frequency, namely $f_D NT$. Figure 6, where $1/r$ vs. ε is plotted for some other values of the block length and for various modulation models, leads to the same conclusion, except perhaps for very short blocks ($N = 100$ symbols in the figure), which may be of limited interest anyway. More specifically, it can be seen that, as long as the block length is not too small and the average error rate is not too high, $1/r$ is relatively constant *for any given value of* ε, i.e., given the first-order statistics, the second-order statistics exhibit limited sensitivity to the coding/modulation scheme used. Also, this behavior is more evident in M-ary FSK with large M.

Also shown in Figures 4–6 are the results obtained for the simple threshold model described in Section III. The match between these results and those for the more detailed simulation model is very good, indicating that all curves given in Figures 4–6 can be approximated by those found for the threshold model (which are much easier to compute). Note that the threshold model as defined in (4) implicitly relies on two fairly strict assumptions. First, the fading process is slow enough so that the channel can be considered constant throughout a block of data, i.e., the sampled version of $\alpha(t)$ at rate $1/NT$, $\alpha(iNT)$, is adequate to represent the fading value for every symbol in the i-th block. Second, the relationship between the instantaneous signal-to-noise ratio and the error probability is close to a step function, i.e., it has a threshold behavior (e.g., when coding is used). One would expect that, when these conditions are not satisfied, the threshold model would be inadequate. Somewhat surprisingly, this turned out not to be the case. The results in this paper show that the threshold model, even in cases when the above conditions are not met, may be used to obtain a "universal curve" which approximates the relationship between $1/r$ and ε in a variety of cases, due to its limited sensitivity to the coding/modulation scheme.

This motivates us to present the analytical expressions of the Markov parameters for the important case of flat Rayleigh fading, which for the threshold model can be found as [6]

$$\varepsilon = 1 - e^{-b} \tag{15}$$

and

$$r = \frac{Q(\theta, \rho\theta) - Q(\rho\theta, \theta)}{e^{b} - 1}, \tag{16}$$

respectively, where

$$\theta = \sqrt{\frac{2b}{1-\rho^2}}, \tag{17}$$

$\rho = K(NT) = J_0(2\pi f_D NT)$ is the correlation coefficient of two samples (spaced by NT seconds) of the complex Gaussian fading process, and

$$Q(x,y) = \int_y^{\infty} e^{-\frac{(x^2+w^2)}{2}} I_0(xw) w dw \tag{18}$$

is the Marcum-Q function [13]. The relationship between the two Markov parameters can therefore be easily found as

$$r = \frac{1-\varepsilon}{\varepsilon} \left[Q(\theta, \rho\theta) - Q(\rho\theta, \theta) \right], \tag{19}$$

where

$$\theta = \sqrt{\frac{-2\log(1-\varepsilon)}{1 - J_0^2(2\pi f_D NT)}}. \tag{20}$$

Equation (19), as already discussed, can be used as an approximation for the functional relationship between the Markov parameters ε and r, regardless of the actual coding/modulation scheme used.

Note that, in order to use this approximate approach, we still need to compute the average block error rate, ε, which of course does depend on the details of the modulation/coding scheme used. On the other hand, unlike the second order error statistics (needed to compute the transition probability, r), the marginal error rate, ε, has been studied in a number of papers (e.g., see [11; 14]). It is therefore possible to use available results for ε and then (19) to identify the transition probability of the error process, r, leading to the complete Markov characterization of the error model. We note in passing that $F_e = -1/\log(1-\varepsilon)$ can be considered as the *equivalent* fading margin, to be used in a threshold model which approximates the error statistics for the scheme under consideration.

Using these results, it is possible to find fairly accurate results in a very simple way, without need for lengthy simulations. As an example, the dependence of the burst length on the Doppler frequency, the block length and the block error rate is studied in Figure 7, which shows $1/r$ vs. the quantity $f_D NT$, i.e., the Doppler frequency normalized to the block transmission rate, for some values of ε. It is interesting to see how, on a logarithmic scale, the curves are very close to straight lines for values of the average length of a burst of errors larger than 2 blocks. Therefore, in the presence of clustered errors, the average burst length can be reduced by a factor of 2 by doubling the Doppler frequency (i.e., the mobile speed or the carrier frequency), or by doubling the block length, or by halving the channel symbol rate. Again, this is intuitively true in the presence of long bursts of errors. The significance of our results here is to show that this behavior is exhibited for short error bursts as well (as short as 2 blocks).

Thus, the Markov approximation, used in conjunction with the threshold model, is a useful tool to describe the data block error process produced by a fading channel, and to simplify the performance evaluation of higher-layer protocols, provided that they are not too sensitive to the Markov parameters.

V. EXAMPLE: THROUGHPUT OF ARQ PROTOCOLS OVER FADING CHANNELS

As an example of the application of the above theory, we consider here the performance analysis of ARQ data-link error control schemes. Results on the computation of the throughput of classic ARQ schemes on Markov channels have been published in the literature [3, Ch. 5], [15]–[20]. These results can be directly used to compute the performance of such schemes in fading channels, where the block errors can be accurately modeled by means of a Markov process, as discussed above.

More specifically, consider the classic Go-Back-N scheme [21]. We define the throughput as the average rate at which blocks are accepted at the receiver. This quantity is, in general, smaller than the success rate on the channel, $1-\varepsilon$, since when a block error occurs, some good blocks may be rejected as well, due to the protocol operation.

Based on the Markov parameters evaluated by simulation as explained above, we computed the throughput performance in three different ways. The first neglects any information on the block error correlation, i.e., it assumes that the errors are described by an iid process, whereas the second takes into account the block error correlation by using a first-order Markov model for the block error process. For simplicity, in this example we assume that no feedback errors occur (throughput analyses accounting for unreliable feedback can be found in [18; 19; 20]). The throughput performance in these two cases is given by [16]

$$\eta = \frac{1-\varepsilon}{1-\varepsilon+m\varepsilon} \qquad \text{(iid)} \tag{21}$$

and

$$\begin{aligned} \eta &= \frac{r(m)}{mq + r(m)} \\ &= \frac{r[1-(p-r)^m]}{r[1-(p-r)^m] + mq(q+r)} \\ &= \left(1 + \frac{r\varepsilon}{1-\varepsilon} \cdot \frac{m(1-\varepsilon)^{m-1}}{(1-\varepsilon)^m - (1-\varepsilon-r)^m}\right)^{-1} \\ &\qquad \text{(first-order Markov)}, \end{aligned} \tag{22}$$

respectively, where m is the round-trip delay in slots. Finally, we simulated the fading process according to the scheme proposed by Jakes [9], and we ran the protocol over such a simulated channel.

Figure 8 shows some numerical results, for BPSK modulation, $f_D NT = 0.02$, and various values of N and n_e. The results corresponding to the threshold model are also plotted for comparison. In each case, analytical results using (21) (iid errors) and (22) (Markov errors) are plotted (in the figure, the curves for iid are indistinguishable from each other), whereas the triangles are the results obtained by direct simulation. It can be clearly seen that the iid model, which neglects the block error correlation, provides a very poor prediction of the performance, whereas the simple first-order Markov model matches the simulation results very closely: the accuracy is very good for larger values of the block size, N, whereas for small N (e.g., $N = 100$ in Figure 8) the analytical model gives slightly conservative results. We also investigated the effectiveness of higher-order Markov models, but we obtained results very close to the first-order model, showing that the memory of one block is sufficient to capture the slowly-varying nature of the channel, and more complicated models are actually not needed.

These results indicate the inadequacy of the simple average block error rate as a measure of the performance, since neglecting the second-order characterization of the errors leads to a gross underestimation of the performance. In fact, the marginal statistics of the error process are independent of one of the key parameters of the fading process, i.e., the Doppler frequency, and therefore provide too simplified a description.

Finally, in Figure 9 we have plotted the throughput results for two values of the parameter $f_D NT$, and for three different modulation techiques, along with the results found from the threshold model and those computed by direct simulation. The relationship between η and ε remains the same, for a given value of $f_D NT$, regardless of the scheme used. In particular, the computationally faster results obtained for the threshold model provide a very accurate estimate of the performance found by following the more detailed approach (for the slow fading case, i.e., $f_D NT = 0.02$, they are indistinguishable from the other results). This further supports our claim that the threshold model can be usefully employed for a simple, but still accurate, performance analysis of data transmission schemes in fading channels.

VI. CONCLUSIONS

In this paper, we have investigated the behavior of block errors which arise in data transmission on fading channels. Our approach takes into account the details of the specific coding/modulation scheme, and tracks the fading process symbol-by-symbol. It is shown that a Markov approximation for the block error process (possibly degenerating into an iid process for sufficiently fast fading) is a very good model for a broad range of parameters.

Also, the Markov parameters for the various communications schemes considered in this paper are compared with those found in [6] for the simpler threshold model. It is observed that the relationship between the marginal error rate and the transition probability is largely insensitive to parameters such as block length, degree of forward error correction and modulation format, and depends only on an appropriately normalized version of the Doppler frequency. This relationship can therefore be simply computed using the threshold model, and then used more generally as an accurate approximation. This approach leads to a simplified performance analysis of upper-layer protocols.

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FIGURE CAPTIONS

Fig. 1. Entropy of the binary process β_i, $H(\beta)$ (solid), average mutual information, $I(\beta_i;\beta_{i-1})$ (dotted), and conditional average mutual information, $I(\beta_i;\beta_{i-2}|\beta_{i-1})$ (dashed), vs. the average block error rate, ε, for $f_DT = 0.0001$ and various values of the block length, N, and of the error correction capability, n_e. Coherent BPSK, $N = 100$, $n_e = 10$ (•), $N = 500$, $n_e = 50$ (○) and $N = 2000$, $n_e = 200$ (□).

Fig. 2. Entropy of the binary process β_i, $H(\beta)$ (solid), average mutual information, $I(\beta_i;\beta_{i-1})$ (dotted), and conditional average mutual information, $I(\beta_i;\beta_{i-2}|\beta_{i-1})$ (dashed), vs. the average block error rate, ε, for $f_DT = 0.0001$. Non-coherent M-ary FSK, $M = 2, 4, 8, 16$ and 32, $N = 500$, $n_e = 50$.

Fig. 3. Entropy of the binary process β_i, $H(\beta)$ (solid), average mutual information, $I(\beta_i;\beta_{i-1})$ (dotted), and conditional average mutual information, $I(\beta_i;\beta_{i-2}|\beta_{i-1})$ (dashed), vs. the average block error rate, ε, for Coherent BPSK, $N = 500$, $n_e = 50$, and for $f_DNT = 1$ (•), $f_DT = 0.1$ (○) and $f_DNT = 0.01$ (□).

Fig. 4. Average burst length, $1/r$, vs. the average block error rate, ε, for $N = 500$, $n_e = 4$ (solid) and 50 (dotted), and for the threshold model (•). Coherent BPSK, $f_DNT = 1$, 0.1, 0.05 and 0.02.

Fig. 5. Average burst length, $1/r$, vs. the average block error rate, ε, for $N = 500$, $n_e = 4$ and 50, and for the threshold model. Non-coherent M-ary FSK, $M = 2$ and 16, $f_DT = 1$ and 0.05.

Fig. 6. Average burst length, $1/r$, vs. the average block error rate, ε, for $N = 100$, $n_e = 10$ (solid), $N = 500$, $n_e = 50$ (dotted), $N = 2000$, $n_e = 200$ (dashed), and for the threshold model (•). Coherent BPSK (a), Noncoherent M-ary FSK with $M = 2$ (b) and $M = 16$ (c), $f_DNT = 0.02$.

Fig. 7. Average burst length, $1/r$, vs. the normalized Doppler frequency, f_DNT, for some values of the error rate: $\varepsilon = 0.01, 0.02, 0.05$ and 0.1; threshold model.

Fig. 8. Average throughput of GBN, η, vs. the average block error rate, ε, for BPSK modulation with various block lengths and coding parameters, and for the threshold model. Markov and iid models compared with the simulation results.

Fig. 9. Average throughput of GBN, η, vs. the average block error rate, ε, for various modulation schemes and for $N = 500$ and $n_e = 50$, and for the threshold model. Markov models compared with the simulation results.

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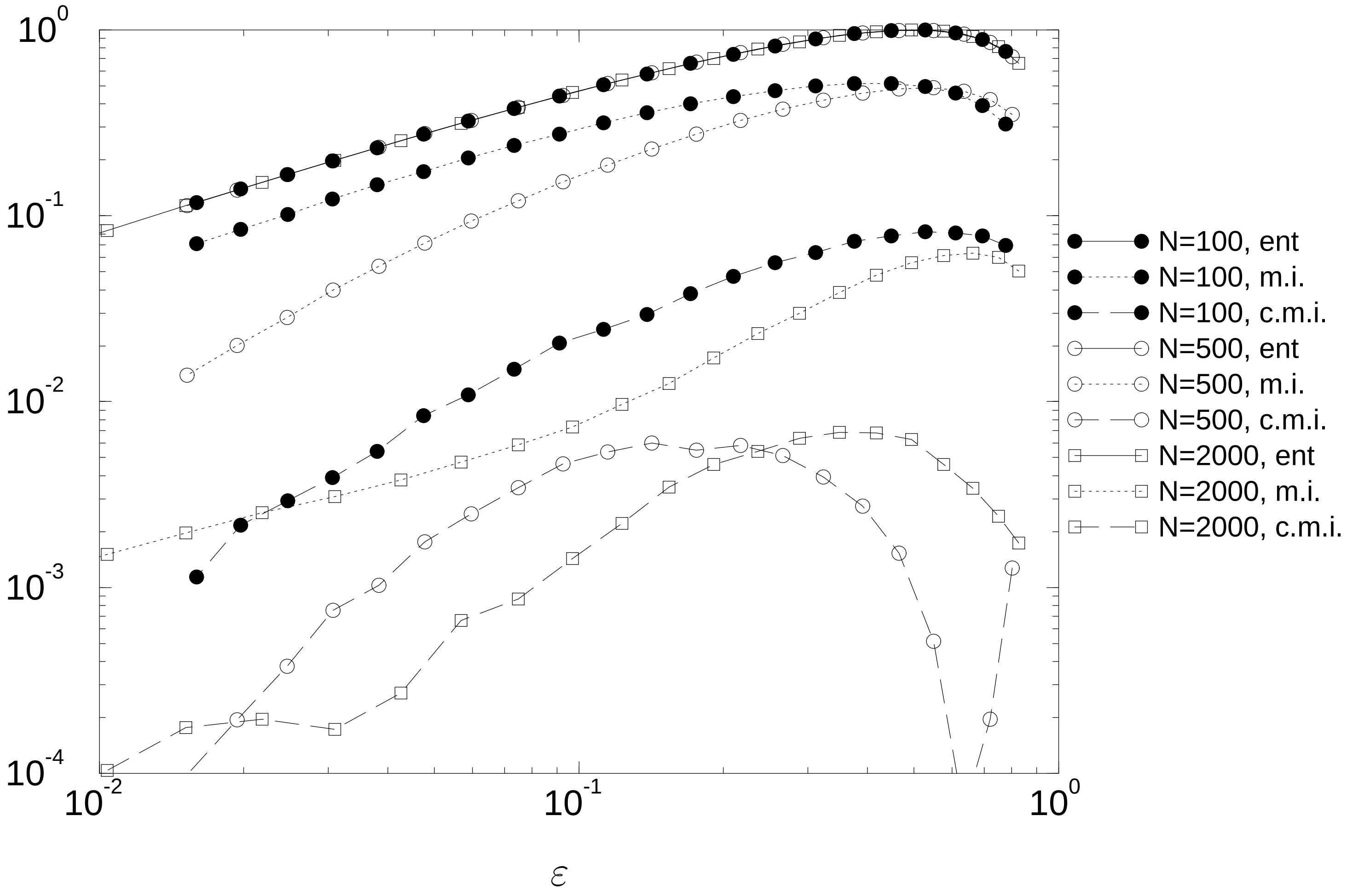

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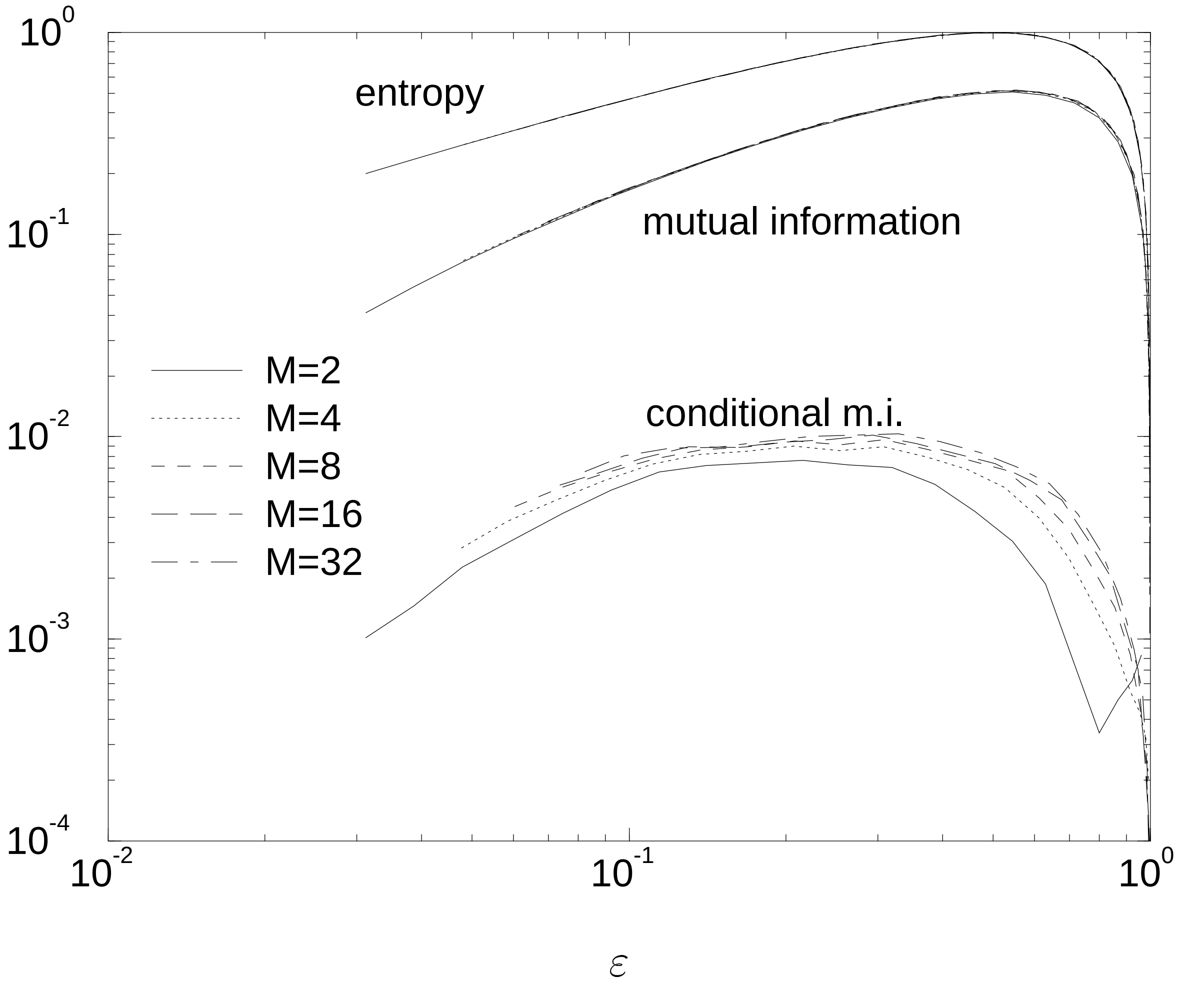

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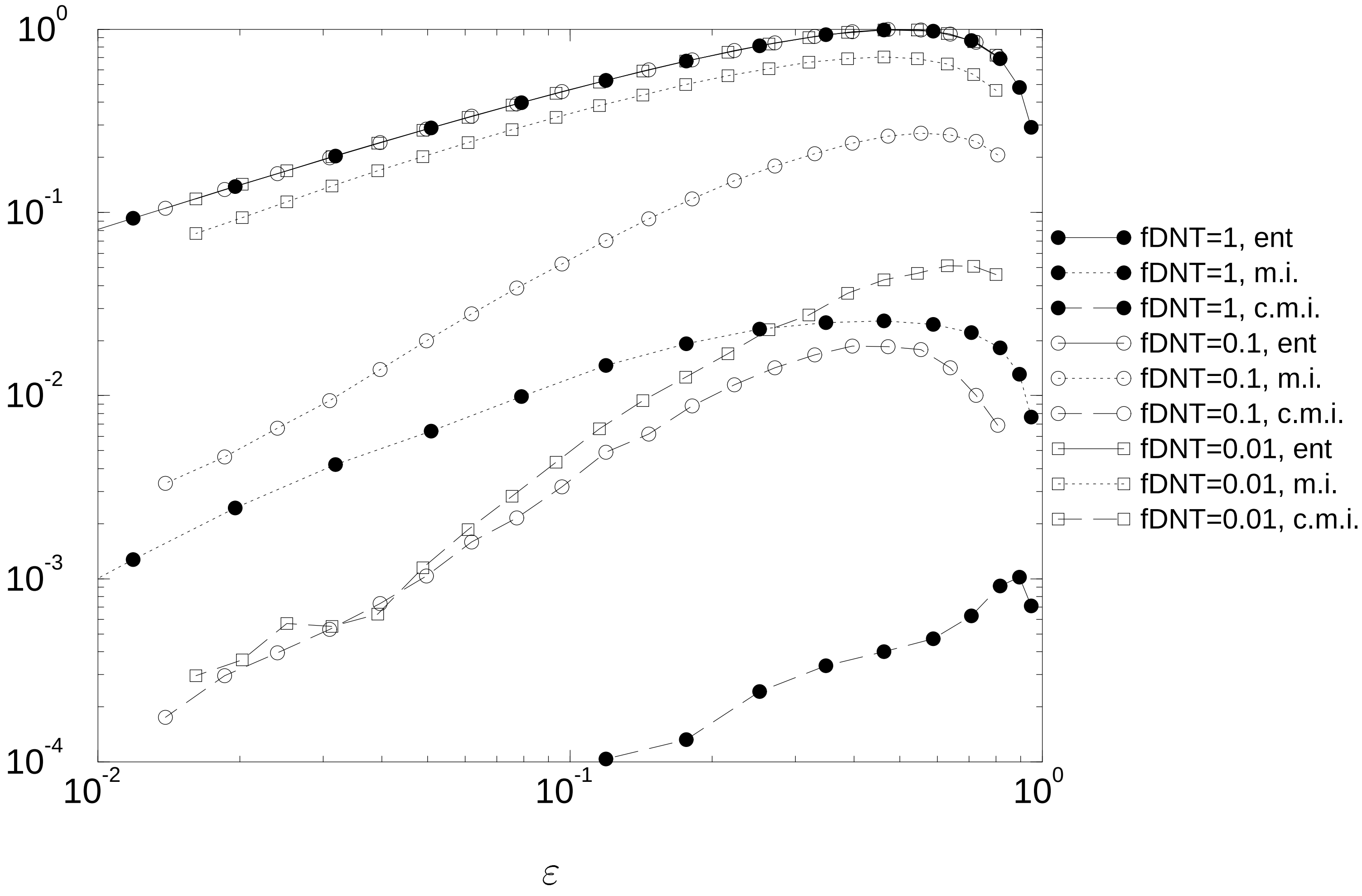

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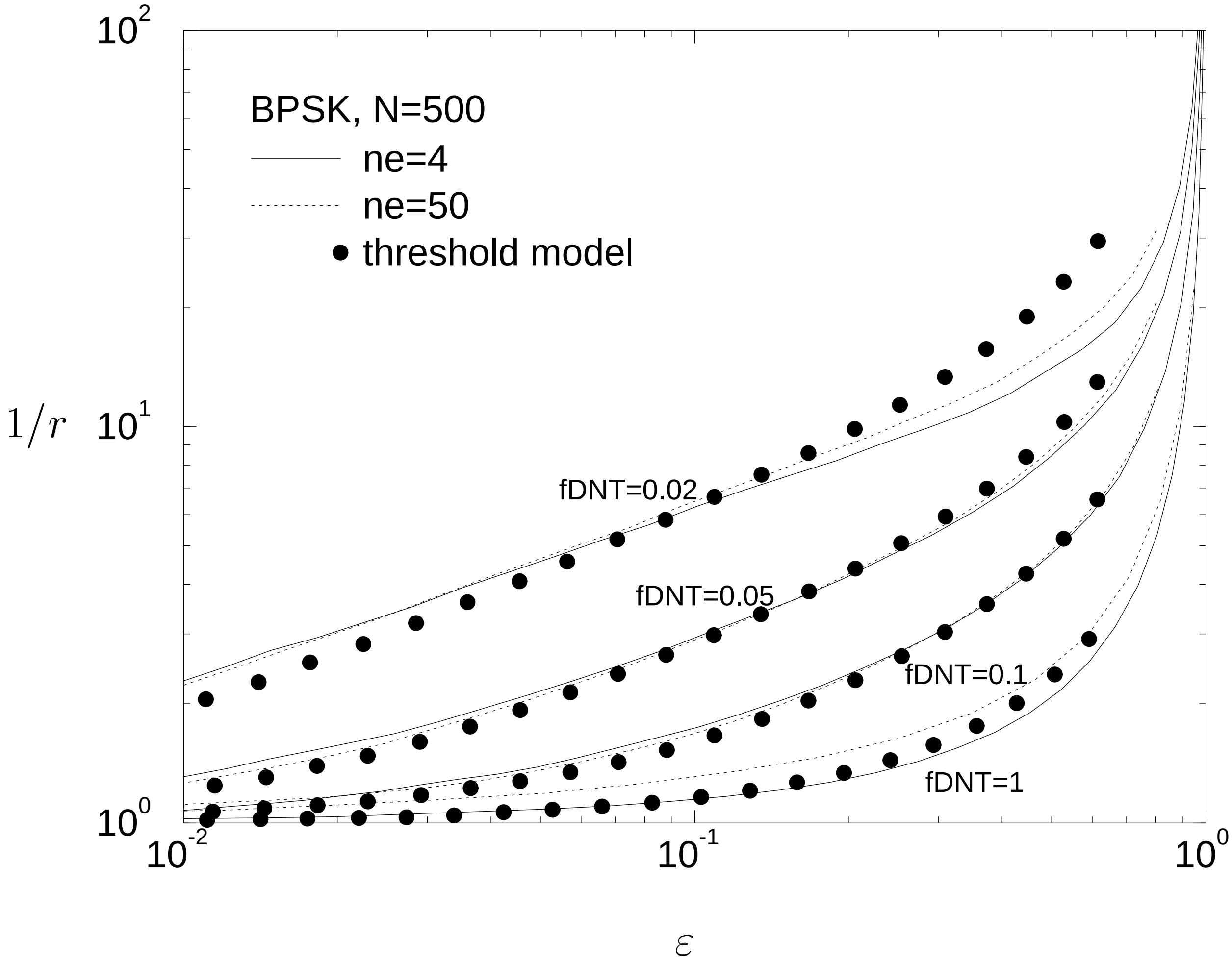

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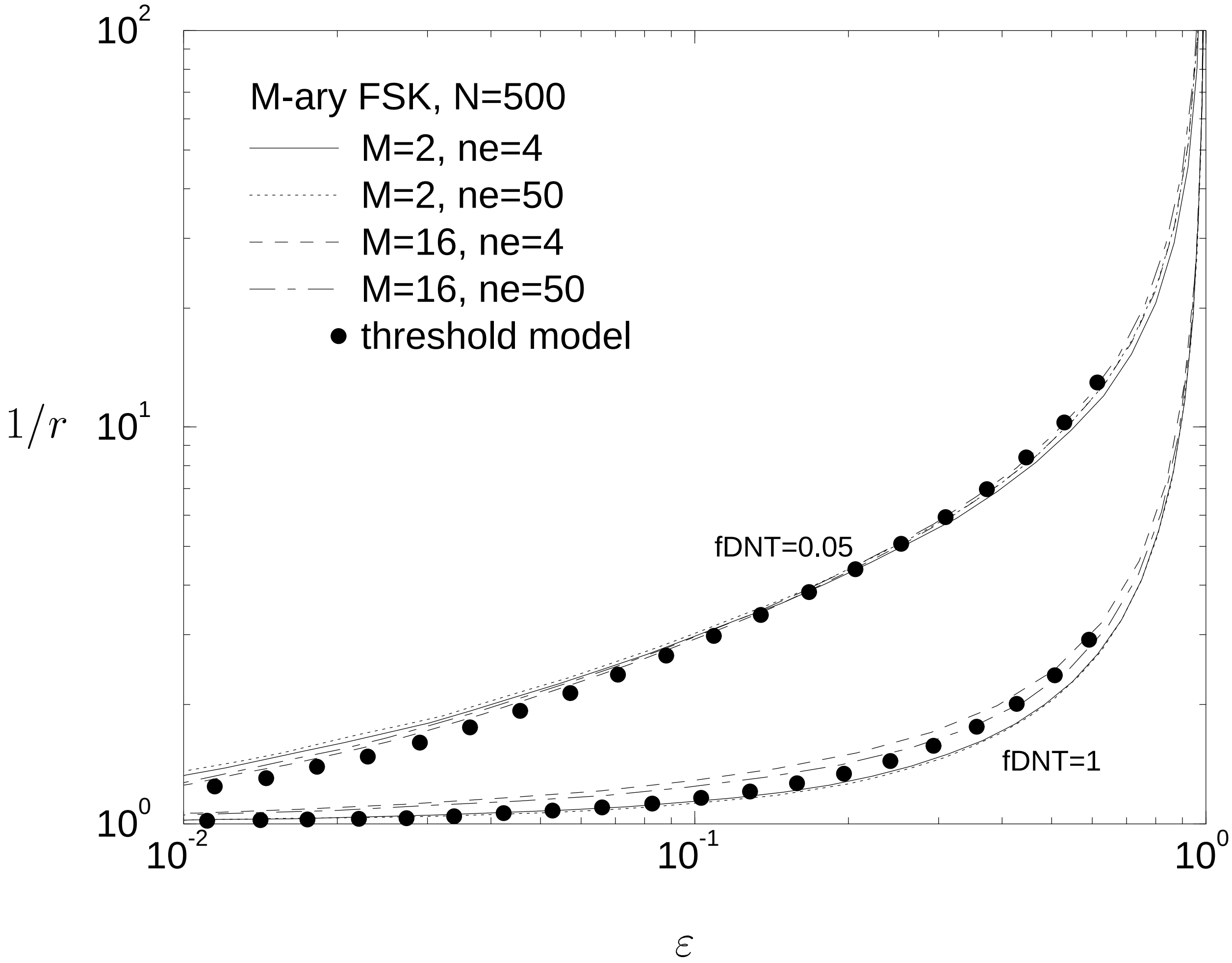

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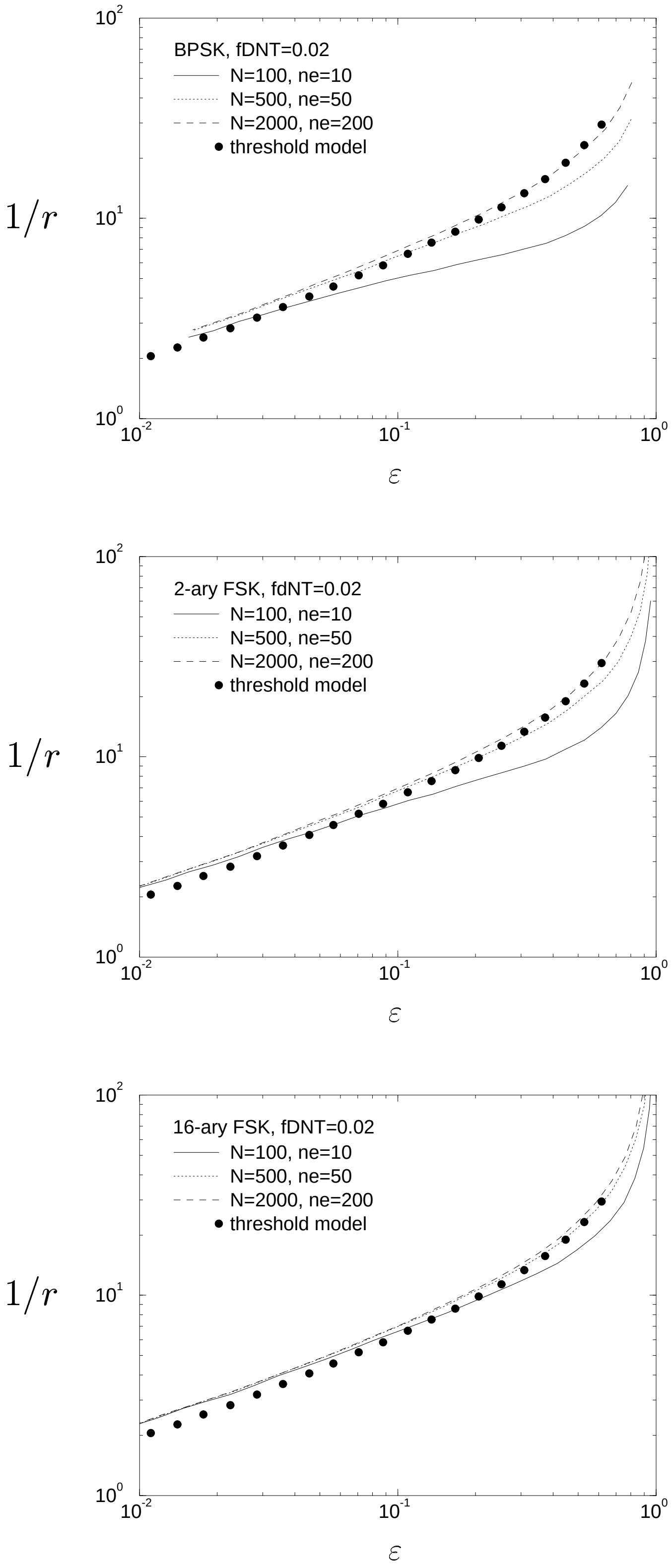
(a)

(b)

(c)



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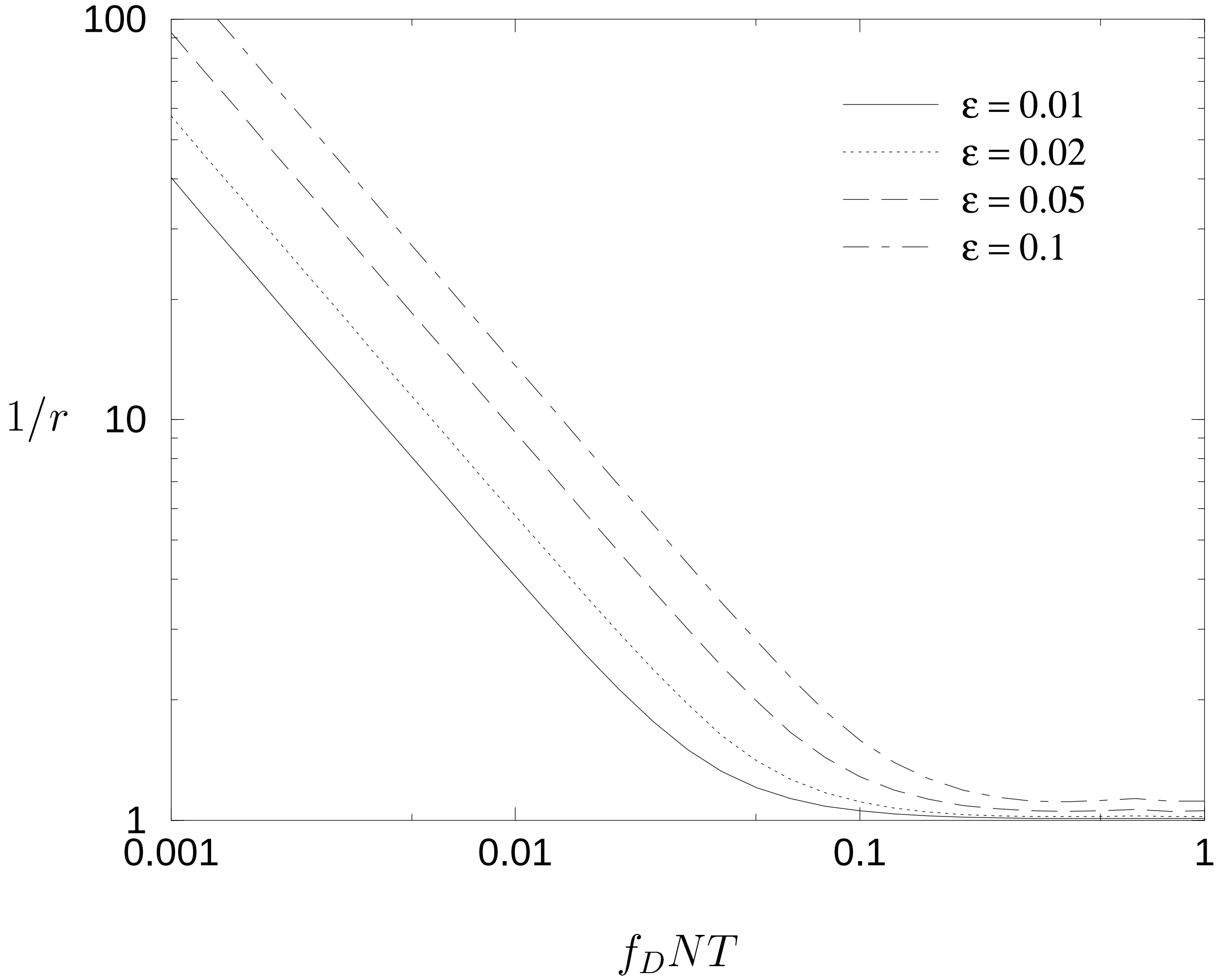

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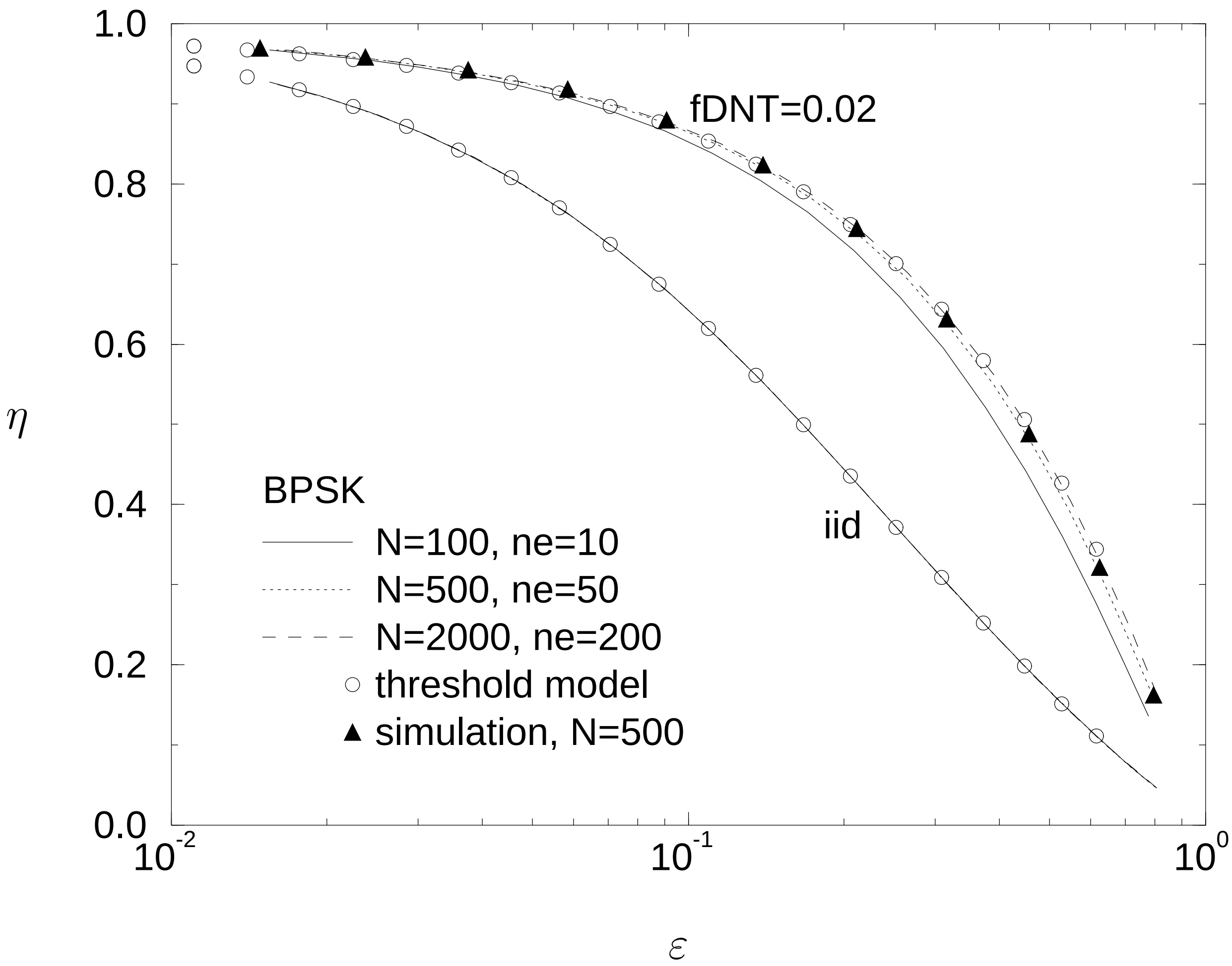

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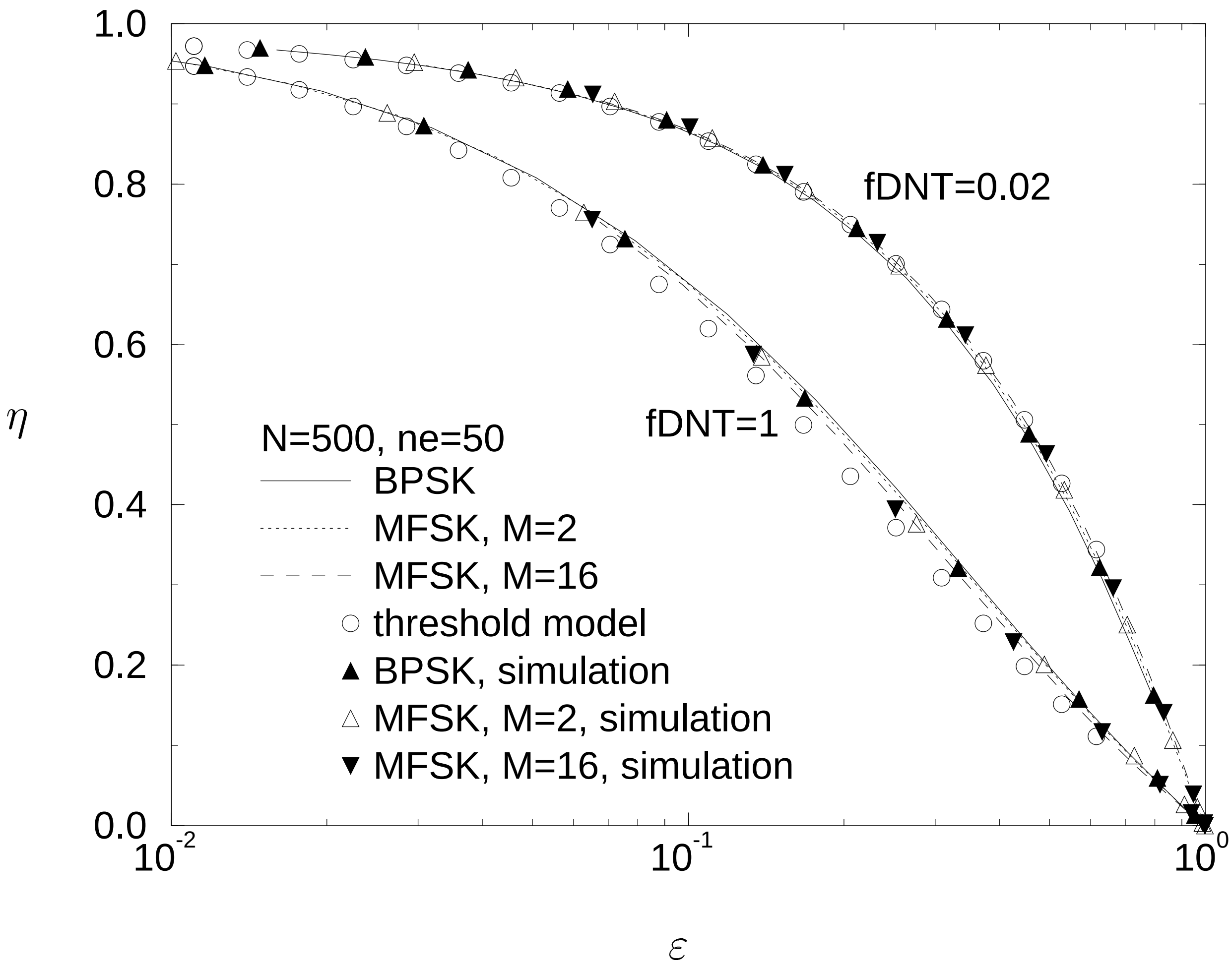

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