Variable Aggregation in the ILP Design of WDM Networks with Dedicated Protection

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***Abstract*—Protection technique is a key factor in WDM networks. In such networks, a link failure may cause the failure of several optical channels, thereby leading to large data loss. Consequently in this work we investigate the issue of planning and optimization of protected WDM networks which has raised much interest in these last years. Integer Linear Programming (ILP) is one of the most common exact method in solving optimization problems of protected WDM networks. In this paper firstly we present a variable aggregation method that has the ability of significantly reducing the computational complexity of the traditional ILP flow formulation. Then we compare the computational burden of flow formulation with variable aggregation both with the classical flow formulation and with the route formulation. The comparison is carried out by applying the three alternative methods to the optimization of two case-study networks.**

I. Introduction

Optical networks based on Wavelength Division Multiplexing (WDM) are the main characters today of the transport infrastructure evolution towards high capacity and high reliability. These networks are based on switching and routing of optical circuits in space and wavelength switching domains. Recently, on the switching equipment side, Optical Cross Connects (OXC) systems have become available, beside the more mature Optical Add-Drop Multiplexers. This opened up the road to the possibility of deploying complex WDM networks based on the mesh topology, while in the past single ring or overlaid multi-ring were the most used architectures for WDM. The increase in WDM complexity brought the need for suitable network planning strategies into foreground. Problems such as optimal routing and resource allocation for optical connections must be continuously solved by new and old operators, to plan new installations or to update and expand the existing ones. These problems can no longer be manually solved in complex network architectures, as it usually happened in the earlier experimental WDM installations. Computer-aided planning tools and procedures are needed for the future which can achieve an efficient utilization of network resources in a reasonable computational time.

In this paper WDM network design is developed in order to guarantee network survivability against a link failure; the issue of survivability of optical connections has become of outstanding importance today: a loss of a high speed connection operating at such bit rates as Gb/s or higher, even for few seconds, means huge waste of data. Undoubtedly protection technique adoption is paid off by a more complex network design: this has to include in the optimization a further term of complexity in order to include the capacity terms needed to reroute optical connections after a link failure.

Research on optical network since some years ago has been investigating design and optimization techniques. The various proposed solutions can be classified into two main groups: heuristic methods and exact methods. The former returns sub-optimal solutions that in many cases are acceptable and have the advantage of requiring a limited computational effort. The latter are much more computationally intensive and do not scale well with the network size, being even not applicable in some cases; however since they are able to identify the absolute optimal solution, they play a fundamental role either as direct planning tools or as benchmarks to validate and test the heuristic methods.

The work we are presenting concerns exact methods to plan and optimize resilient multifiber WDM networks. In particular we focus on Integer Linear Programming (ILP), a widespread technique to solve exact optimization. In [1] we have proposed a new formulation of the optimization problem, called *source formulation*, which allows a relevant computational weight reduction. Unfortunately source formulation can not be extended to protected WDM networks; in this article we focus our attention on path protection strategy: first we present two traditional ILP approaches, then we propose an alternative and more scalable model to obtain the results with less computational effort.

The rest of the paper is organized as follows. In section II we introduce our network model and present a short review of the literature regarding ILP application to WDM path protected network optimization. In section III the path-protected formulations are presented and explained into details. We investigated dedicated case, while the shared case analysis can be found in [2], [3]. In section IV we discuss a possible simplification of the traditional flow formulation applying a variables aggregation: this is possible thanks to the imposition of a necessary condition. Finally, in section V results obtained by applying the formulations to case-study networks are shown; using the case study then we are able to point out the advantages of the model changes we are proposing

II. Resilient WDM network optimization by integer linear programming

Network design and planning is carried out with different techniques according to the type of traffic the network has to support. We investigate the static traffic case in which a known set of permanent connection requests is assigned *a priori* to the network. The connections requested by the nodes at a given time to a WDM network all together form the offered traffic matrix (alias virtual topology). Each request is for a point-to-point optical circuit (lightpath) able to carry a given capacity from the source optical termination to the destination termination and each node pair may request more than one connection.

Connections must be established by suitably configuring network switching resources and allocating network transmission resources: a lightpath, when established, is a sequence of WDM channels, one for each fiber it crosses. We assume that all the WDM channels carry the same capacity. Lightpaths are routed and switched by the OXCs of the network and the two lightpath terminations are located in the source and the destination OXCs. The channels composing the lightpath may have different wavelengths or may be all at the same wavelength, according to the availability of the wavelength conversion function in the transit OXCs. To simplify, we consider only Virtual Wavelength Path (VWP) network cases, in which all the OXC's are able to perform full wavelength conversion (i.e. an incoming optical signal having any wavelength can be converted to an outgoing optical signal having any possible transmission wavelength); on the contrary in Wavelength Path (WP) network case no wavelength conversion is allowed in the whole network and lightpaths are subjected to the "wavelength continuity" constraint, that is absent in the VWP case.

WDM network today are often designed in order to be resilient to failures that may occur to switching or transmission equipment. This study examines path protection, a well-known approach to survive single link (or node) failures in a optical network: for each connection a backup path is statically reserved along with a working path between the source and destination nodes during call setup. In dedicated path protection (also called 1+1 protection), each primary path has a dedicated link-disjoint backup path. In shared-path protection, a link-disjoint backup path of a given connection can share WDM channels reserved for spare lightpaths associated to other connection requests.

Static optimization of a WDM network can be so summarized: given a static traffic matrix, find the optimum values of a set of network variables that minimizes a given cost (or objective) function, under a set of constraints. Since optical WDM network design is essentially an optimization problem, its exact solution, corresponding to the absolute minimum of the cost function, can be found by applying linear programming; then the fact that capacity is expressed in terms of number of WDM channels leads to the integer constraint on the variables (ILP). The choice of variables, cost function and constraints greatly varies from case to case. The work we are proposing follows a current trend in which virtual topology optimization is accompanied by cost minimization of a multi-fiber physical network: the number of fibers per link is a variable of the problem to be minimized, while the amount of wavelengths per fiber is usually preset [4].

Finding the minimal network capacity that allows to route all the connections is the first result of the planning problem. The second goal of planning is the allocation of physical resources to the lightpaths that have to be set up, i.e. the definition of the sequence of WDM channels composing each lightpath. As a whole, solving static planning corresponds to solve RFWA, routing, fiber, wavelength assignment, i.e. to find the optimal resources assignment for a preassigned traffic matrix over a preassigned physical topology.

WDM network optimization by ILP has been widely studied in literature and in the following we focus our attention on ILP formulations in path protected scenarios. We can subdivide research contributions in two groups according to which type of networks they are applied to:

- WDM networks with single-fiber links;
- multifiber WDM networks.

In the first group the problem consists in optimal routing and wavelength assignment (RWA) of the lightpaths. This is a NP-complete problem, as it was demonstrated in Refs. [5], [6]. Two basic methods has been defined to model the RWA problem: *flow formulation* and *route formulation* [7]. In the former the basic variables are the flows on each link relative to each source-destination OXC pair (or connection requests); in the latter the basic variables are the paths connecting each source-destination pair. Both these two formulations have been employed to solve various sorts of problems and to investigate different aspects of WDM networks. Solving RWA problem has been often associated to survivability matters: in [8] a complete description of the different protection strategies is shown using ILP models.

In optimization of multifiber WDM networks optimal allocation of fibers has also to be solved, thus complicating the problem of lightpath set up into routing, fiber and wavelength assignment (RFWA). Solving RFWA becomes really challenging even with relatively small networks, especially because routing and wavelength assignment is coupled to dimensioning. In this case a new set of variables representing the number of fibers of each physical link must be considered in addition to the flow or the route variables defined above for the two corresponding formulations. This implies that RFWA scales from a multicommodity flow problem to a more complex localization problem. The protection issue is taken into account also in works facing RFWA problem: in [9] an ILP model for path protection is presented; also in [10] path protection is studied under the different hypothesis of dedicated and shared backup paths; another exhaustive analysis of protection strategies based on ILP models can be found in [11], where link and path protection are described.

When the problem becomes computationally impractical, a typical simplification strategy is to impose routing constraint. For example, all the lightpaths can be constrained to be routed along the first s shortest paths connecting the source to the destination. In this cases route formulation becomes more useful than flow formulation since its solution complexity can be reduced. Differently from the flow formulation, the complexity of which is strictly dependent on physical and virtual topologies, the size of the route formulation decreases with the number of paths that can be employed to route the lightpaths. Multifiber network optimization with route formulation and constrained routing has been studied in Refs. [12], [9], [10], [7]. Beside route formulation with constrained routing, other methods to control complexity have been proposed. A possibility is to stop the branch-and-bound algorithm (typically used to solve ILP problems) after having found the first or a pre-definite number of integer solutions. Ref. [11] shows that acceptable results (though quite far from the optimal solution) can be obtained when the branch-and-bound duration is fixed to 10 minutes. Ref. [13] proposed that the whole RFWA problem can be solved in a sequence of simpler problems (e.g. first routing, then fiber assignment, and so on). Other possible approaches are: exploitation of lagrangean relaxation [14], relaxation of integer constraint [12] and randomized routing [15].

Moreover a large body of research is available in the Operations Research literature regarding ILP resolution algorithms that exploit efficient methods such as cutting plane or cut-set inequalities [16], [17], [18]. These proposals are alternative to the traditional branch-and bound method to solve capacitated network design (it's worth noting that RFWA in VWP network , i.e. without wavelength continuity constraint, is the same as a capacitated network design). Finally, similar studies on the capacitated network design associated to path restoration requirements have been carried out for the ATM networks [19]. In the following, our objective will not be the development of an "ad hoc" method to solve RFWA problem with protection requirements, but we will focus on the analysis of effects of an opportune variable aggregation in order to reduce the problem size and repercussions on optimality and computing performance.

Undoubtedly the massive need of computational resources (i.e. time and memory occupation) represents the main obstacle to an efficient application of ILP in optical networks design. As we have shown in the preceding literature overview, route formulation with constrained routing or other simplification techniques are able to overcome this limitation. The most of them produces only approximations of the actual problem optimum. On the other hand, the great advantage of ILP over heuristic methods is the ability to *guarantee* that the obtained solution is the absolute optimum value (e.g. for benchmarking purpose).In the following we present an efficient flow-based ILP formulation developed to solve planning of WDM networks exploiting dedicated path protection as survivability strategy. This formulation shows good performances in design problem with large offered traffic load compared to classical route and flow models. Although it relaxes the traditional set of link-disjointness constraints, it returns the optimal values in all the realistic cases we take into account. Moreover we propose a simple algorithm to verify if the obtained solution is the problem absolute optimum (i.e. it is not affected by approximations), so that a not-admissible solution could be identified

III. ILP MODELS IN DEDICATED PATH PROTECTED NETWORKS

Let us consider a multifiber WDM network environment under static traffic, in which the number of wavelengths per fiber W is given *a priori*, while the number of fibers installed in each physical link are variables of the problem.

ILP allows to deal with resilient routing problem by a *one step* approach, jointly assigning working and spare paths under the hypothesis of edge disjoint path protection. Most of the other previously mentioned heuristic or approximated techniques, based on multistep methods, can not guarantee the optimality of the solution. The definition of an ILP model in a WDM network with dedicated path protection is a well-known problem: to the usual set of constraints used in the unprotected network [1], we must add constraints deriving from the link disjointness condition deriving from dedicated path protection strategy. These additional constraints can be easily set exploiting the traditional flow or route variables. This, however, will result in a computationally heavy representation of the problem, since we need to distinguish each connection requests in order to protect the requests one by one, thus involving a large number of variables.

Our aim is to investigate an alternative ILP formulation able to simplify the complex dimensioning problem. We propose the "Max Half" formulation that soothes the computational burden of the traditional flow formulation model by collapsing all connection requests relative to the same source-destination couple in a single variable.

Now let us explain and compare the flow, route and "Max Half" formulations into details. We consider a VWP network, provided with full wavelength conversion as defined in II. The extension of ILP models from VWP to WP case can be carried on as explained in [7], [1], but it introduces a further term of complexity in dependence of the value W the number of wavelengths supported by each fiber.

A. Flow Formulation

The physical topology is modeled by the graph $\mathcal{G} = \mathcal{G}(\mathcal{N}, \mathcal{A})$ [1]. Physical links are represented by the undirected edges $l \in \mathcal{A}$ with $|\mathcal{A}| = L$, while the nodes $i \in \mathcal{N} = \{1, 2, ...N\}$, with $|\mathcal{N}| = N$, represent the OXCs. Each link is equipped with a certain amount of unidirectional fibers in each of the two directions; fiber direction is conventionally identified by the binary variable k ($k = 0$ for forward direction, $k = 1$ for backward direction). Each source destination node couple requiring connectivity (lightpaths) is associated to an index c. We refer to the source node as s_c and to destination as d_c; the required traffic is v_c: if $v_c > 1$, we add an auxiliary index t having values between

[1]All the following formulations require that the topology is at least 2-connected

1 and v_c [2]. Connection requests are unidirectional. As far as dimensioning and resource allocation are concerned, it is not relevant to fix a distinction between the working and the protection lightpath associated to the same connection request. Therefore we will refer to a 1+1 protected optical connection in terms of a link-disjoint couple of paths connecting the source node to the destination node.

Let us define all the variables involved in this protected flow formulation:

- $x_{l,k,c,t}$ is a boolean variable indicating whether a WDM channel on link l on a fiber having direction k has been allocated to the t-th connection requested by node couple c.
- $F_{l,k}$ is the number of fibers on link l in direction k.

The following additional symbols are also defined:

- (l,k) identifies the set of fibers of link l that are directed as indicated by k; in the following we name (l,k) a "unidirectional link";
- I_i^+ is the set of "unidirectional links" having the node i as one extreme and leaving the node; analogously, I_i^- is the set of "unidirectional links" having the node i as a one extreme and pointing towards the node;
- (c,t) identifies a single connection request: c identifies the connection source-destination couple, while t identifies one particular connection request associated to the node couple c.

Now we can detail the flow formulation. The cost function to be minimized can be either the total fiber number

$$\min \sum_{(l,k)} F_{l,k}$$

or alternatively can contain an estimation of the cost $W_{l,k}$ of link (l,k) ($\sum_{(l,k)} W_{l,k} \cdot F_{l,k}$). We refer to this second metric as length metric, while the first is called hop metric.

The set of constraints is the following

$$\sum_{(l,k)\in I_i^+} x_{l,k,c,t} - \sum_{(l,k)\in I_i^-} x_{l,k,c,t} = \begin{cases} 2 & \text{if } i = s_c \\ -2 & \text{if } i = d_c \\ 0 & \text{otherwise} \end{cases} \qquad \forall\ i,(c,t); \qquad (1)$$

$$\sum_{(c,t)} x_{l,k,c,t} \le W \cdot F_{l,k} \qquad \forall\ (l,k); \qquad (2)$$

$$\sum_{k} x_{l,k,c,t} \le 1 \qquad \forall\ l,(c,t); \qquad (3)$$

$$x_{l,k,c,t} \quad \text{binary} \qquad \forall\ (l,k),(c,t); \qquad (4)$$

$$F_{l,k} \quad \text{integer} \qquad \forall\ (l,k); \qquad (5)$$

This formulation assigns a routing with respect to dedicated path protection strategy, as described in II.

[2]Indices c and t could collapse in a single index directly associated to each single connection request, but this alternative notation is less intuitive. In the following we will use the former indexing for sake of clarity.

Constraint (1) is a solenoidality constraint. It corresponds to the following sequence. Let us consider the t-th connection requested by node couple c. We express the flow conservation condition for each node i of the network, considering only traffic associated to connection (c,t). This condition states that the total flow (c,t) leaving i must be equal to the total flow c incident on i. This equation is slightly modified in the source (destination) node of the connection request (c,t), in which the outgoing (incoming) flow must be equal to 2. This is due to the fact that two lightpaths (working+spare) are associated to the connection request, according to the dedicated path protection technique. Constraints concerning dimensioning are a simple extensions of the corresponding constraints in the unprotected case. Constraint (2) ensures that the total number of WDM channels allocated to spare and working lightpaths on the unidirectional link (l,k) is bounded by the link capacity, given by the number of fibers $F_{l,k}$ multiplied by the number of wavelength W. Constraint (3) stems from link-disjointness condition: no more than one lightpath associated to connection request (c,t) can coexist on the same link, neither in opposite direction. Let us note that (3) prevents the coexistence of two working (or spare) flows on opposite directions on the same link; this is a reasonable constraints: in fact such a situation would be associated to an useless allocation of a cycle (anyway ILP optimization would avoid this kind of routing). From now on for sake of simplicity we refer to these formulations with the acronym FF (Flow Formulation).

B. Route Formulation

In order to apply route formulation, we have to carry out a preprocessing operation to prepare the set of route variables for the ILP optimization[3]. Preprocessing is carried out only for the node couples which require connections. All the link-disjoint cycles connecting the source-destination nodes are identified. A cycle is a pair of two link-disjoint routes connecting the nodes through the network. Cycle identification is performed assuming that the network is completely idle of traffic and that each link has unlimited capacity: that is, a WDM channel is always available between two nodes provided that a physical link exists. We have assigned a variable to every working-spare couple and not to every single path to make route formulation more competitive: if we assign a variable to every single path we should use a constraint similar to (3) used in the previous flow formulation. So the approach to route formulation that we propose allows us to reduce both variables and constraints number.

Let's consider a source-destination couple c and suppose we have precomputed all the n working-spare routes between this two nodes. We can then identify a working-spare route using index (c,n). The variable $r_{c,n}$ indicates how many protected connections are routed on the n^{th} working-spare route between node couple c. The subset $\mathcal{R}_{(l,k)}$ includes all the working-spare routes whose working is routed on link (l,k). The objective

[3]Let's now observe that preprocessing time can not be neglected in any case. We will clarify this aspect in the last section

function is the same seen in the FF model. Let's analyze the constraints.

$$\sum_{n} r_{c,n} = v_c \qquad \forall\ c; \tag{6}$$

$$\sum_{(c,n)\in\mathcal{R}_{(l,k)}} r_{c,n} \leq WF_{l,k} \qquad \forall\ (l,k); \tag{7}$$

$$r_{c,n} \text{ integer} \qquad \forall\ (c,n);$$

$$F_{l,k} \text{ integer} \qquad \forall\ (l,k);$$

Constraint (6) ensures that number of working-spare routes established between each source-destination couple c satisfies offered load v_c. Constraint (7) ensures that number of fiber on link (l,k) can support working and spare traffic routed on this link. From now on for we refer to these formulations with the acronym RF (Route Formulation).

IV. Setting a necessary condition: the "Max Half"(MH) per link

In the previous section we have reviewed two well-known ILP models in dedicated path protected network case; unfortunately the application of these models to real networks cases is affected by computational limitations. The number of admissible paths in a mesh network grows rapidly for increasing values of connectivity index and number of nodes; route approach becomes unfeasible if we do not introduce the constrained routing. On the other hand the flow approach is not scalable on the volume of offered traffic because it requires a large variable number especially if offered traffic matrix contains a considerable number of connections. But this last limitation of the flow model could be removed taking note of a simple consideration: the variable number can be reduced *by aggregating all the flow variables associated to a given node couple*, i.e $\sum_t x_{l,k,c,t} = x_{l,k,c}$. Using this substitution we can obtain a more efficient model, paying just a relaxed description of protection mechanism. Let us try to explain this last statement. It's immediate to verify that, if we route a doubled traffic (i.e. working+spare) and we assume not to concentrate more than the total number of connections requested by each node couple on a single link (i.e. half traffic), automatically the obtained routing would be able to deliver all the offered traffic in case of link failure. This is due to the fact that for each node pair a link failure cannot waste more than a half of routed paths and so a number of paths sufficient for traffic delivery is left in the network (that's why we call this approach *Max Half*)[4]. This property is a necessary condition to obtain a routing satisfying link-disjointness between working and spare lightpath: if more than half traffic associated to a node pair is gathered on a single link, it will be impossible to separate in link-disjoint paths the working and the spare traffic. Unfortunately this condition is not sufficient: in Fig. 1 we show a case of routing assignment which satisfies the MH (no more than 2 lightpaths on each link), but it does not verify the link-disjointness. Between nodes A and B we have routed 2 protected connection requests (so 4 lightpaths in total). These lightpaths are routed so that a link failure does not preclude the delivery of at least 2 lightpaths. In order to obtain an edge-disjoint path protected routing:

- in case of link z failure, p_1 and p_2 have to be working lightpath , while p_3 and p_4 spare;
- in case of link x failure, p_3 and p_2 have to be working lightpath, while p_1 and p_4 spare;
- in case of link y failure, p_3 and p_1 have to be working lightpath , while p_2 and p_4 spare.

In conclusion, this model guarantees a sufficient number of paths to survive in order to deliver requested traffic in case of link failure. Anyway, this is a necessary condition; it is not sufficient to guarantee the property of the link-disjointness of routing (an example is shown in Fig. 1).

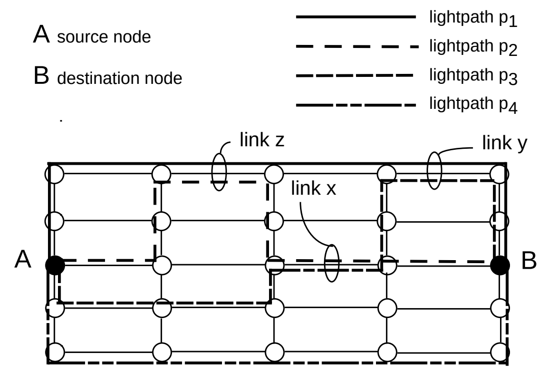

Fig. 1. This routing assignment satisfies the necessary condition imposed by ""Max Half"" formulation, but it does not satisfies link-disjointness.

Nevertheless this model remains interesting: optimal routings analogous to that shown in Fig. 1 are associated to very particular network scenarios (presumably scenarios that could appear when the transmission resources are given as an input of the problem, not in a dimensioning case). As a matter of fact we have conducted optimization runs on two well-known case-study networks and on a set of 8-node wheel networks with increasing values of connectivity index, always assuming different values of W, and MH have always returned a link-disjoint routing assignment. In conclusion MH allows us to obtain optimal values on our network cases showing better performance with respect to flow and route formulations.

To verify routing link-disjointness, we suggest to use the algorithm for maximum matching in a graph: for each node couple that requires connections, let's construct a graph, so that its nodes are associated to the paths resulting by optimization; two nodes are connected if they do not share any links. If there exist an optimum matching in the resulting graph, then the routing satisfies the property of link-disjointness. Otherwise

[4] In other words, MH guarantees lightpath restoration, i.e protection against link failures in a network where the subdivision between working and spare lightpath could be dynamically updated depending on the failed link; this is obtained by imposing the survivability of a sufficient number of paths for traffic delivery. Clearly this ILP approach can not be applied in a real restoration scenario due to the strict time constraints in restoration management.

the not-disjoint lightpath must be rerouted, trying to minimize the additional needed capacity.

Let us analyze the constraints of the new model :

$$\sum_{(l,k)\in I_i^+} x_{l,k,c} - \sum_{(l,k)\in I_i^-} x_{l,k,c} = \begin{cases} 2\cdot v_c & \text{if i} = s_c \\ -2\cdot v_c & \text{if i} = d_c \\ 0 & \text{otherwise} \end{cases} \qquad \forall\ i,c; \qquad (8)$$

$$\sum_c x_{l,k,c} \leq WF_{l,k} \qquad \forall\ (l,k); \qquad (9)$$

$$\sum_k x_{l,k,c} \leq v_c \qquad \forall\ l,c; \qquad (10)$$

$$x_{l,k,c} \text{ integer} \qquad \forall\ (l,k),c;$$

$$F_{l,k} \text{ integer} \qquad \forall\ (l,k);$$

Solenoidality constraint routes a doubled traffic; as in the previous formulation we refer with a unique variable to working and spare traffic (8). Constraints (9) do not change significantly compared to unprotected case. Constraint (10) sets the necessary condition previously discussed: no more than half of the traffic (i.e. v_c because in the protected case the total traffic is $2\cdot v_c$) can flow on the same link in both backward or forward direction. In the following we refer to this model with the acronym MH ("Max Half").

We can now compare the complexity of the presented models, in terms of number of constraints and variables. The following notation will be used:

- N number of nodes;
- L number of bidirectional links;
- W number of wavelengths per fiber;
- T total offered traffic (in terms of number of protected connections requests);
- C number of nodes couples requiring connections;
- R average number of working-spare paths (cycles) of a node couple.

Table I compares the complexity of FF, RF and DP formulation. Let's now observe the difference between FF and DF, the flow based formulations. "Max Half" allows us to save variables and constraints in the order of T/C, that is to say that performance improvement will tend to be more significant for heavy traffic loads. In the NSFNET case study the traffic matrix is characterized by $C = 108$ (node pairs requiring connections) and $T = 360$ (total connection requests), so the DP gain is approximately 3,3 on both variables and constraints number. The comparison with RF depends on R value and we will discuss it in the next section.

V. Case study and result comparison

In this section we present and discuss the results obtained by performing ILP optimization exploiting FF, RF and DP formulation on case-study networks. Two well-known networks have been considered: the National Science Foundation Network (NSFNET) and the European Optical Network (EON). Data

TABLE I

Comparison on constraint and variable numbers between FF, RF and MH formulations.

formulation	variables	constraints
FF	$2L(T+1)$	$2L+T(N+L)$
MH	$2L(C+1)$	$2L+C(N+L)$
RF	$R\cdot C+2L$	$C+2L$

regarding their physical topologies, were taken from Ref.[10] and Ref.[20], respectively. NSFNET has 14 nodes and 22 links, while EON has 19 nodes and 39 links. The static (symmetric) traffic matrices are derived from real traffic measurements which are reported in the same references and they comprise 360 and 1380 unidirectional connection requests for NSFNET and EON, respectively.

To solve the ILP problems we used the software tool CPLEX 6.5 based on the branch-and-bound method [21]. As hardware platform a workstation equipped with a 1 GHz processor was used. The available memory (physical RAM + swap) amounted to 460 Megabyte. This last parameter plays a fundamental role in performing our optimization. The branch-and-bound algorithm progressively occupies memory with its data structure while it is running. When the optimal solution is found, the algorithm stops and the computational time and the final memory occupation can be measured. In some cases, however, all the available memory is filled up before the optimal solution can be found. In this cases CPLEX returns the best but non-optimal branch-and-bound solution it has been able to find and forces the execution to quit. This cases are identified by the *out-of-memory* tag (O.O.M.) and the computational time measures how long it has taken to fill up memory. We have clarified this particular aspect of ILP to allow to clear understanding of the reported data.

Table II shows the number of variables and constraints that are involved in the ILP problem applied to the two networks in the VWP case. Data are taken from the parameters returned by CPLEX presolver. They clearly show the advantage of DP formulation on FF (in the order of T/C as foreseen). The advantage of DP on RF can be observed in the EON case, while in the NSFNET the complexity of the two models is comparable. RF based problems become intractable due to the exponential growth of admissible paths with mesh network complexity, while flow variables grow linearly with the link number. As a matter of fact the preprocessing time in the NSFNET case takes about an hour and half, while in the EON case we give up precalculating all the admissible paths after having computed all the routes between a single source-destination node couple obtaining about 80000 paths (i.e variables) in 2 days of computational time (but the number of node couples requiring connections is 342!). The number of variables reported in Table II is an estimation ($342\cdot 80000$ variables).

We have shown the advantage of FF versus MH in terms of variable and constraint numbers. It is important to see how much this advantage impacts on the actual computational

TABLE II

ILP VARIABLES AND CONSTRAINTS FOR NSFNET AND EON.

network/formul.	constraints	variables
NSFNET/FF	10856	13726
NSFNET/DP	3932	4796
NSFNET/RF	152	22468
EON/FF	67122	94104
EON/DP	19396	25980
EON/RF	420	$\approx 27 \cdot 10^6$

TABLE III

COMPUTATIONAL TIME AND MEMORY OCCUPATION COMPARISON BETWEEN FF AND MH FORMULATIONS IN NSFNET VWP NETWORK.

W	FF	MH
2	20m	1.5m
4	2h	3m
8	41.8h	5.4h
16	4.4h	1.5h
32	33.7h	43m
64	80h	3.4h

W	FF	MH
2	0.62MB	0.26MB
4	3MB	0.54MB
8	OOM	OOM
16	55MB	73MB
32	354MB	20MB
64	250MB	30MB

performance of ILP. Tables III display computational times and memory occupations of NSFNET optimization in the VWP case (s, m, and h stand for seconds, minutes and hours respectively, while MB stands for Megabyte). There is a great difference in terms of computational time: in table III it is shown an example concerning NSFNET, with T/C=3.3: for example in W=2,4 cases in MH model the computational time has order of magnitude of second; in FF model the order is minutes or hours. Analogous consideration can be expressed on Table IV in the EON case.

The main advantage of MH over FF model is the aggregation of traffic associated to a source-destination pair: this aggregation allows us to use integer variables instead of binary variables so reducing the number of variables by a factor (T/C). So, in heavily loaded networks, with large values of (T/C), the advantages of MH are more evident, in terms of computational time, memory occupation and quality of the solution found[5]. In order to highlight the substantial performance difference between the two formulations, we have carried on a sequence of optimizations runs on NSFNET by increasing the offered traffic load. The original traffic matrix, with T/C=3.3, is multiplied by a factor γ, with $\gamma = 10, 20, 30, 40$. The different behavior of FF and MH formulation on NSFNET with $W = 4$ is summarized in Table V. Results confirm that MH

[5]The two models may give different solutions only when the B & B algorithm doesn't terminate, i.e. when the entire memory saturates.

TABLE IV

COMPUTATIONAL TIME COMPARISON BETWEEN FF AND MH FORMULATIONS IN EON NETWORK.

W	FF	MH
2	20m	10s
4	25.6h	6.2h
8	41.4h	11.6h
16	29.9h	9.2h
32	77.2h	28.5h
64	76h	25h

TABLE V

COMPUTATIONAL TIME AND MEMORY OCCUPATION COMPARISON BETWEEN FF MH AND RF FORMULATION IN NSFNET VWP NETWORK, W=4.

γ	FF	MH	RF
10	38m - 200 MB	9s - 9 MB	15s - 11 MB
20	8h - 400 MB	5s - 6 MB	11s - 9 MB
30	Can't find integer solution	4s - 6 MB	6s - 7 MB
40	Can't read problem data	5s - 5 MB	15s - 9 MB

model is independent of the traffic increment: optimization runs on NSFNET with $W = 2, 4$ show that MH is always able to find the optimal solution in less than 10 seconds, by occupying a limited amount of memory. Also with $\gamma = 100$, MH provides the same performance. On the contrary, the runs with the FF model confirm that the traffic increment has a wasting effect on problem complexity and consequently on FF performance. Execution time and memory occupation increase considerably. We need 200 Megabyte and some minutes when $\gamma = 10$ and 400 Megabyte and some hours when $\gamma = 20$. When $\gamma = 30$ it become impossible to find any feasible integer solution; when $\gamma = 40$ it becomes impossible to read problem data. For sake of completeness, an analogous analysis has been carried out using route formulation to show that also RF does not depend on traffic load. Anyway the efficiency comparison between RF and MH must be set from another point of view. Although RF complexity does not depend on traffic load (as seen for FF), its main drawback is related to the exponential relation between network dimension and the number of admissible paths pair between each node couple. For example upgrading network complexity from 14 nodes and 22 links in NSFNET case to 19 nodes and 39 links in EON case we have observed in Table II that RF approach becomes intractable, while MH approach provides good performances despite the increase in network dimension.

A. Results with path protection dedicated strategy

We have tried to analyze the two case-study networks under dedicated path-protection exploiting FF, RF and MH models. MH model succeeds in finding optimum or at least values very close to optimal value and outperforms FF and RF models from a computational point of view. Let's now analyze the values obtained in the two topologies.

We run optimizations in NSFNET and EON networks varying W from 2 to 64: in NSFNET final values are optimal for each W except for $W = 16$, where the partial result returned after memory exhausting is characterized by a per cent distance from optimum not greater than 1% (we use the *gap* parameter contained in CPLEX). In EON solution optimality is verified only with $W = 2$, but percent error of other solutions is lower than 1% for $W = 4, 8, 16$, 1,6% for $W = 32$ and 2,6% for $W = 64$; these approximations seem to be acceptable and we suppose that a bigger availability of computational resources would have allowed us to demonstrate that the obtained integer solutions coincide with problems optima. In Fig.2 and Fig.3 we show results compared with

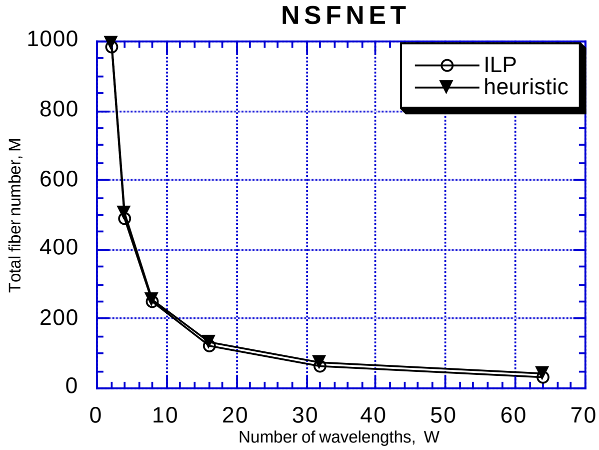

Fig. 2. Total fiber number in NSFNET network with dedicated path protection.

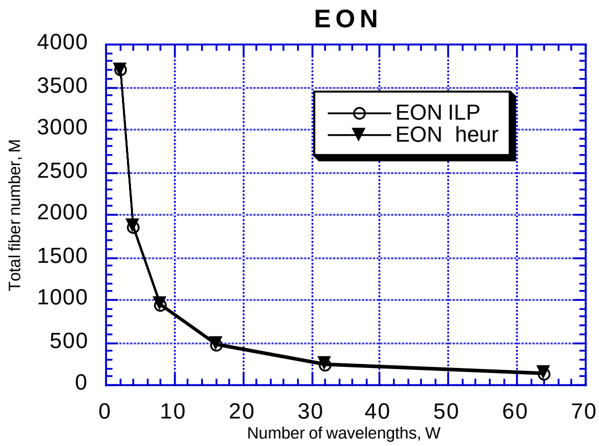

Fig. 3. Total fiber number in EON network with dedicated path protection.

the numerical values obtained by the heuristic tool ([22]) developed by our research group. Thanks to MH efficiency we are able to benchmark our heuristic tool in the two case studies in a reasonable time. Indeed for small network ILP performances can be considered competitive with heuristic tool performances.

VI. Conclusions

We have considered the problem of designing and optimizing WDM resilient multifiber networks supporting unidirectional protected optical connections. We have presented Max Half formulation (MH), a novel approach to model the above problem with low computational complexity in the particular case of static traffic and dedicated path protection strategy. Thanks to the MH formulation, we are able to substantially reduce the multiplicity of both variables and constraints compared to the traditional flow formulation especially when traffic matrix is very large. A comparison has been carried out also with the well-known route formulation showing that it is out performed by MH in large networks (e.g. the EON). Exploiting MH we thus obtain a substantial gain with respect to the traditional route and flow models with significantly low computational times and memory occupations.

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